Cosmological predictions from the Misner brane

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Within the spirit of five-dimensional gravity in the Randall-Sundrum scenario, in this paper we consider cosmological and gravitational implications induced by forcing the spacetime metric to satisfy a Misner-like symmetry. We first show that in the resulting Misner-brane framework the Friedmann metric for a radiation dominated flat universe and the Schwarzschild or anti-de Sitter black hole metrics are exact solutions on the branes, but the model cannot accommodate any inflationary solution. The horizon and flatness problems can however be solved in Misner-brane cosmology by causal and noncausal communications through the extra dimension between distant regions which are outside the horizon. Based on a semiclassical approximation to the path-integral approach, we have calculated the quantum state of the Misner-brane universe and the quantum perturbations induced on its metric by brane propagation along the fifth direction. We have then considered testable predictions from our model. These include a scale-invariant spectrum of density perturbations whose amplitude can be naturally accommodated to the required value 10^{-5} - 10^{-6}, and a power spectrum of CMB anisotropies whose acoustic peaks are at the same sky angles as those predicted by inflationary models, but having much smaller secondary-peak intensities. These predictions seem to be compatible with COBE and recent Boomerang and Maxima measurements.

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I. INTRODUCTION

Extra dimensions have a long history in gravitational physics since Kaluza [1] and Klein [2] first introduced a fifth coordinate in Einstein general relativity to account for a unified description of graviy and electromagnetism. Although string and superstring theories have reached a great deal of progress using and extending the Kaluza-Klein idea, the issue of extra dimensions has never stirred theorists so much as some work carried out by Horava, Witten, Randall and Sundrum has done recently. Working in the realm of the ten-dimensional $E_8 \times E_8$ heterotic string theory, Horava and Witten first showed [3] that this theory can be related to an eleven-dimensional theory on the orbifold $\mathbf{R}^{10} \times \mathbf{S}^1/\mathbf{Z}_2$ in such a way that whereas the standard model particles are confined to the four-dimensional spacetime, gravitons propagate in the full bulk space. By simplifying the Harova-Witten framework to five dimensions, Randall and Sundrum then derived [4,5] two very interesting models where there exist two branes placed on the extra direction (which are identical to two domain walls with oposite tensions) in five-dimensional anti-de Sitter spacetime. In their first model [4], they assumed that we live in the negative-tension brane and proposed a mechanism to solve the hierarchy problem based on a small extra dimension. This scenario may have the serious problem that gravity is repulsive in the brane with negative tension [6]. Although this conclusion might be the subject of debate, one at least can certainly say that a negative tension on the physical brane brings some interpretational difficulties to the model. The second Randall-Sundrum model assumed [5] that we live in the positive-tension brane, while the other brane is moved off to infinity, so localizing gravity in one of the two three-branes only. The problem with this scenario is that the field equations in the positive-tension brane are nonlinear in the source terms [7].

Thus, the second Randall-Sundrum model avoids any possible physical effect from repulsive gravity, or at least negative brane tension in the observable universe, but unfortunately leads to noncoventional (i.e. non Friedmann) cosmology [7,8] when matter in the positive-tension brane is isotropically and homogeneously distributed, or non-conventional chargeless, nonrotating black holes [9] when the matter in that brane is assumed to collapse without rotating beyond its trapped surface. Again some controversy can arise concerning the cosmological difficulties of the second Randall-Sundrum model - in particular it could be thought that the nonlinear source terms are all irrelevant at any era at which we are able to do cosmology. However, a pefectly standard Friedmann evolution appears to be a most desirable property for any cosmological model. In spite of the several attempts made in order to reconcile the nonconventional cosmological bihaviour with standard Friedmann scenario based on inserting a cosmological constant in the brane universe [10-13], within the spirit of the Randall-Sundrum approach [4,5], or on reinterpreting black-hole physics in the brane world [9], it appears that the cosmological and collapse scenarios resulting from any of the two Randall-Sundrum models may have serious difficulties when comparing them with current observations or theoretical requirements.

On the other hand, several authors have obtained in-

flationary solutions for the three-brane worlds with nontrivial configurations in the extra dimension [11,14-16]. Most of such solutions are based on imposing different absolute values for the tensions of the two branes, taking the resulting neat tension as the source of the exponential expansion [15]. Of course, the implicit main aim of all these inflationary brane models is at solving standard cosmological puzzles such as horizon and flatness problems. It is in this sense that inflationary mechanisms operating in brane worlds may however be regarded as superfluous. In fact, it has been shown by Chung and Freese [17] that signals traveling along null geodesics on the extra dimension in Randall-Sundrum spacetimes actually connect distant points which otherwise are outside the horizon, so solving as well the horizon and, eventually, the flatness problems and whereby making unnecessary inflation as a cosmological ingradient of the brane worlds. Recently, a new approach which in a way can be considered as a combination of the two randall-Sundrum models, has been also suggested [18] to solve the above potential shortcomings of such models. Given a five-dimensional spacetime with two domain walls with oposite tensions placed on the fifth coordinate, in order to suitably represent the universe we live in the new approach chooses neither of the two branes individually, but both of them simultaneously by imposing Misner symmetry and whereby allowing nonchronal regions with closed timelike curves (CTC's) only in the bulk.

The resulting cosmological scenario placed on the two-brane system is then quite conventional: it just describes a standard Friedmann universe in the radiation dominated era. After exploring further the gravitational physics of this scenario, it is also shown in the present paper that such a scenario can quite naturally accommodate Schwarzschild or anti-de Sitter black holes by simply allowing matter in the branes to collapse without rotating, but not any kind of inflationary mechanism. Once the conventional causally-generated gravitational behaviour is recovered for matter in the branes, this paper aims at exploring cosmological predictions from Misner-brane universe by: (i) investigating the noninflationary connection mechanisms between distant points (which otherwise are outside the horizon) that can solve the horizon and flatness problems, (ii) formulating the quantum state of the Misner-brane universe by resorting to the semiclassical approximation of the Euclidean path-integral formalism, (iii) deriving the spectra of initial density perturbations and CMB anisotropies arising from quantum fluctuations induced on the branes by propagating along the fifth direction. We regard as the main results of the paper the predictions of a scale-independent spectrum for primordial density perturbations whose amplitude can easily accommodate the requirement that the density contrast be smaller or approximately equal to 10^{-5} [19], and of a power spectrum for CMB anisotropies which seems to fit recent data by Boomerang [20] and Maxima [21] experiments better than any other proposed models.

We outline the rest of the paper as follows. In Sec. II we review and extend the results of Ref. [18], where the Misner-brane model was introduced, and discuss some of its physical implications, including spacetime propagation of quantum fields, a calculation of the value of the Planck mass and some solutions of the Klein-Gordon equation for small gravitational fluctuations. The spherically symmetric collapse of neutral matter in the branes which leads to conventional Schwarzschild or anti-de Sitter black holes is studied in Sec. III. Sec. IV deals with the quantum state of the Misner-brane universe. We estimate it by resorting to the semiclassical approximation to the Euclidean path-integral formalism. It is obtained that the probability for this universe increases as the parameters that determine the amplitude of the matter-density fluctuations decrease. The quantum effects induced on the brane spacetime by propagating them along the fifth coordinate are estimated in Sec. V using as well a semiclassical approximation to the path integral. The results from this calculation are then employed to analyse the possible solution to the above-alluded cosmological puzzles, and to obtain and discuss the spectra of primordial density fluctuations and CMB anisotropies. Finally, we summarize and conclude in Sec. VI.

II. MISNER-BRANE COSMOLOGY

In this section we shall first review the spacetime structure of the Misner-brane universe such as it was introduced in Ref. [18], adding then some new material referred to the comparation of such a spacetime with that of the original Randall-Sundrum models and the physical consequences that may be drawn from it. Let us start with a five-dimensional spacetime with the fifth dimension, ω, compactified on S^1, with $-\omega_c \leq \omega \leq \omega_c$, and satisfying the orbifold symmetry $\omega \leftrightarrow -\omega$. On the fifth direction there are two domain walls, with the brane at $\omega = 0$ having positive tension and that at $\omega = \omega_c$ having negative tension. In order to represent the universe we live in, we choose neither of the two branes on ω individually, but both of them simultaneously; that is to say, we shall provide the fifth coordinate with a periodic character, in such a way that the branes at $\omega = 0$ and $\omega = \omega_c$ are identified with each other, so that if one enters the brane at $\omega = 0$, one finds oneself emerging from the brane at $\omega = \omega_c$, without having experienced any tension. If we then set the brane at $\omega = 0$ into motion toward the brane at $\omega = \omega_c$ with a given speed v, in units of the speed of light, our space would resemble five-dimensional Misner space [22], the differences being in the spatial topology and in the definition of time and the closed-up extra direction which would also contract at a rate v. Then, time dilation between the two branes would inexorably lead to the creation of a nonchronal region which will start forming at the future of a given chronology horizon [23].

A. Brane spacetime with Misner symmetry

We shall first consider the metric of the five-dimensional spacetime in terms of Gaussian coordinates centered e.g. on the brane at $\omega = 0$. If we assume the three spatial sections on the branes to be flat, then such a metric can be written in the form [24]

$$ds^2 = c^2(\omega,t)\left(d\omega^2 - dt^2\right) + a^2(\omega,t)\sum_{j=2}^{4} dx_j^2, \qquad (2.1)$$

where if we impose the orbifold condition $\omega \leftrightarrow -\omega$ [4,5,24], the scale factors c and a are given by

$$c^2(\omega,t) = \frac{\dot{f}(u)\dot{g}(v)}{\left[f(u)+g(v)\right]^{\frac{2}{3}}}, \quad a^2(\omega,t) = \left[f(u)+g(v)\right]^{\frac{2}{3}}, \qquad (2.2)$$

with $u = t - |\omega|$ and $v = t + |\omega|$ the retarded and advanced coordinates satisfying the orbifold symmetry, where we have absorbed some length constants into the definition of t and ω, and the overhead dot denotes derivative with respect to time t. If no further symmetries are introduced then $f(u)$ and $g(v)$ are arbitrary functions of u and v, respectively [24] . However, taking metric (2.1) to also satisfy the (Misner) symmetry [25,26]

$$(t,\omega,x_2,x_3,x_4) \leftrightarrow$$

$$(t\cosh(n\omega_c) + \omega\sinh(n\omega_c), t\sinh(n\omega_c) + \omega\cosh(n\omega_c),$$

$$x_2, x_3, x_4)\,, \qquad (2.3)$$

where n is any integer number, makes the functions $f(u)$ and $g(v)$ no longer arbitrary. Invariance of metric (2.1) under symmetry (2.3) can be achieved if we choose for $f(u)$ and $g(v)$ the expressions $f(u) = Z_u \ln u + Y_u$, $g(v) = Z_v \ln v + Y_v$, where the Z's and Y's are arbitrary constants. For the sake of simplicity, throughout this paper we shall use the simplest choice

$$f(u) = \ln u, \quad g(v) = \ln v. \qquad (2.4)$$

Imposing symmetry (2.3) together with the choice for the scale factors given by expressions (2.4) fixes the topology of the five-manifold to correspond to the identification of the domain walls at $\omega = 0$ and at $\omega = \omega_c$ with each other, so that if one enters one of these branes then one finds oneself emerging from the other.

The periodicity property on the extra direction can best be explicited by introducing the coordinate transformation

$$\omega = T\sinh(W), \quad t = T\cosh(W), \qquad (2.5)$$

with which metric (2.1) becomes

$$ds^2 = \frac{\left(\frac{\dot{T}^2}{T^2} - \dot{W}^2\right)}{\ln^{\frac{2}{3}} T^2}\left(T^2 dW^2 - dT^2\right) + \ln^{\frac{2}{3}} T^2 \sum_{j=2}^{4} dx_j^2. \qquad (2.6)$$

Although now metric (2.6) and the new coordinate $T = \sqrt{t^2 - \omega^2}$ (which is timelike provided that $\ln T \geq const. \pm W$) are both invariant under symmetry (2.3), the new extra coordinate W transforms as

$$W \equiv \frac{1}{2}\ln\left(\frac{t+|\omega|}{t-|\omega|}\right) \leftrightarrow W + n\omega_c \qquad (2.7)$$

under that symmetry. On the two identified branes making up the Misner-brane universe, we can describe the four-dimensional spacetime by a metric which can be obtained by slicing the five-dimensional spacetime given by metric (2.6), along surfaces of constant W, i.e.

$$ds^2 = -\frac{\dot{T}^2}{T^2 \ln^{\frac{2}{3}} T^2} dT^2 + \ln^{\frac{2}{3}} T^2 \sum_{j=2}^{4} dx_j^2, \qquad (2.8)$$

which will be taken throughout this paper to describe the spacetime of the universe we live in. On the surfaces at constant $T = T_c$, the resulting four-dimensional metric,

$$ds^2 = -\frac{dW^2}{\sinh^2 W \ln^{2/3} T_c^2} + \ln^{2/3} T_c^2 \sum_{j=2}^{4} dx_j^2,$$

keeps a Lorentzian signature even when we rotate to the consistent Euclidean sector $W = i\Omega$ because on that sector time t is still real (see later on).

The energy-momentum tensor for the brane universe will now have the form:

$$T_i^k = \frac{\delta(\omega - n\omega_c)}{c_b}\mathrm{diag}\left(-\rho, p, p, p, 0\right), \quad n = 0,1,2,3..., \qquad (2.9)$$

where $c_b \equiv (t, \omega = n\omega_c)$. This tensor should be derived using the Israel's jump conditions [27] that follow from the Einstein equations. Using the conditions computed by Binétruy, Deffayet and Langlois [7] and the metric (2.6) we then [24] obtain for the energy density and pressure of our Misner-brane universe:

$$\rho = -\frac{4T\dot{W}}{\kappa_{(5)}^2 \ln^{\frac{2}{3}} T^2 \left(|\dot{T}^2 - T^2\dot{W}^2|\right)^{\frac{1}{2}}} \qquad (2.10)$$

$$p = \frac{2T\dot{T}^2 \ln^{\frac{2}{3}} T^2}{\kappa_{(5)}^2 \left(|\dot{T}^2 - T^2\dot{W}^2|\right)^{\frac{5}{2}}} \frac{d}{dt}\left(\frac{T\dot{W}}{\dot{T}}\right) - \frac{1}{3}\rho. \qquad (2.11)$$

Thus, both the energy density ρ and the pressure p, defined by expressions (2.10) and (2.11), respectively, identically vanish on the sections W=const. Therefore, taking the jump of the component (ω,ω) of the Einstein

equations with the orbifold symmetry [7], one gets on the identified branes

$$\frac{\dot{a}_b^2}{a_b^2} + \frac{\ddot{a}_b}{a_b} = \frac{\dot{a}_b \dot{c}_b}{a_b c_b}, \tag{2.12}$$

where $a_b \equiv a(t, \omega = n\omega_c)$, with $n = 0, 1, 2, 3...$, is the scale factor in our Misner-brane universe.

The breakdown of arbitrariness of functions $f(u)$ and $g(v)$ imposed by symmetry (2.3) prevents the quantity c_b to be a constant normalizable to unity, so the right-hand-side of Eq. (2.12) can be expressed in terms of coordinates T, W as:

$$\frac{\dot{a}_b \dot{c}_b}{a_b c_b} = -\frac{1 + \frac{1}{3\ln T}}{3T^2 \cosh^2 W \ln T}. \tag{2.13}$$

A simple dimensional analysis (performed after restoring the constants absorbed in the definitions of t and ω in Eqs. (2.2)) on the right-hand-side of Eq. (2.13) indicates that if this side is taken to play the role of the source term of the corresponding Friedmann equation, then it must be either quadratic in the energy density if we use $\kappa_{(5)}^2 = M_{(5)}^{-3}$ (with $M_{(5)}$ the five-dimensional reduced Planck mass) as the gravitational coupling, or linear in the energy density and pressure if we use $\kappa_{(4)}^2 = 8\pi G_N = M_{(4)}^{-2}$ (with $M_{(4)}$ the usual four-dimensional reduced Planck mass) as the gravitational coupling. Since $\kappa_{(4)}^2$ should be the gravitational coupling that enters the (Friedmann-) description of our observable four-dimensional universe, we must choose the quantity in the right-hand-side of Eq. (2.13) to represent the combination $-\kappa_{(4)}^2(\rho_b + 3p_b)/6$ which should be associated with the geometrical left-hand-side part of Eq. (2.12) of the corresponding Friedmann equation, when the term proportional to the bulk energy-momentum tensor $T_{\omega\omega}$ is dropped by taking the bulk to be empty. We have then,

$$\rho_b + 3p_b = \frac{2\left(1 + \frac{1}{3\ln T}\right)}{\kappa_{(4)}^2 T^2 \cosh^2 W \ln T}. \tag{2.14}$$

The four-dimensional metric (2.8) can be expressed as that of a homogeneous and isotropic universe with flat spatial geometry, $ds^2 = -d\eta^2 + a(\eta)_b^2 \sum_{j=2}^4 dx_j^2$, if we take for the cosmological time $\eta = 3a(\eta)_b^2/(4\cosh W) = 3\ln^{2/3} T^2/(4\cosh W)$. In this case, the scale factor $a(\eta)_b$ corresponds to that of a radiation dominated flat universe, with $\cosh W = const$ expressing conservation of rest energy, and $p_b = \rho_b/3$ at sufficiently small η. In fact, for small η, it follows then from Eq. (2.14)

$$\rho_b \equiv \rho_b(T, \eta) \simeq \frac{4}{3\kappa_{(4)}^2 T^2 \cosh^2(W) a(\eta)_b^6},$$

or

$$\rho_b(\eta) = a(\eta)_b^2 T^2 \cosh^2 W \rho_b(T, \eta) \simeq \frac{3}{32\pi G_N \eta^2},$$

when expressed in terms of the cosmological time η only.

At least in the noninflationary versions of the Randall-Sundrum models, the tensions on the two branes are assumed to be given by [4,5] $V_{\text{visible}} = -V_{\text{hidden}} = \rho_0 = -p_0$, where ρ_0 and p_0 are the energy density and pressure induced from the bulk on the brane at $y = 0$ ($\omega = 0$). In the case of the Misner-brane world, $\rho_0 = p_0 = 0$, and hence $V_{\text{visible}} = V_{\text{hidden}} = 0$, so when one enters one brane one finds oneself emerging from the other brane without having experienced any tension. In this case, we have however a nonzero "effective" tension which can be defined as $V \propto \rho_b > 0$. On the other hand, Csáki et *al.* have found [10] that any problems arising from having a brane with negative tension disappear in a radiation dominated universe, even in the Randall-Sundrum models.

Having shown that the Misner-brane cosmology based on ansatz (2.4) matches the standard cosmological evolution in the radiation dominated era, we turn now to investigate the nonchronal character of the spacetimes described by metric (2.6). Nonchronal regions in such spacetimes can most easily be uncovered if we redefine the coordinates entering this metric, such that $Y = W - \ln T$ and $\Theta = T^2$. In terms of the new coordinates, the line element (2.6) reads:

$$ds^2 = -\frac{\left(\dot{Y}^2 + \frac{\dot{Y}\dot{\Theta}}{\Theta}\right)}{\ln^{\frac{2}{3}} \Theta} \left(\Theta dY^2 + dY d\Theta\right) + \ln^{\frac{2}{3}} \Theta \sum_{j=2}^{4} dx_j^2. \tag{2.15}$$

This metric is real only for $\Theta > 0$ in which case Y is always timelike if $\dot{Y} > 0$. One will therefore [26] have closed timelike curves (CTC's) only in the bulk, provided $\Theta > 0$, $\dot{Y} > 0$. There will never be CTC's in any of the branes, that is the observable universe.

On the other hand, Singularities of metrics (2.6), (2.8) and (2.15) will appear at $T = 0$ and $T = 1$. The first one corresponds to $\omega = t = 0$, and the second one to $\eta = 0$, the initial singularity at $Y = W$, $t^2 = 1 + \omega^2$, in the radiation dominated universe. We note that the source term $-\kappa_{(4)}^2(\rho_b + 3p_b)/6$ given by Eq. (2.14) also diverges at these singularities. The geodesic incompleteness at $T = 1$ can be removed in the five-dimensional space, by extending metric (2.6) with coordinates defined e.g by $X = \int dW/\ln^{\frac{1}{3}} T^2 - 3\ln^{\frac{2}{3}} T^2/4$, $Z = \int dW/\ln^{\frac{1}{3}} T^2 + 3\ln^{\frac{2}{3}} T^2/4$. Instead of metric (2.6), we obtain then

$$ds^2 = \frac{2}{3}(Z - X)\times$$

$$\left\{\exp\left[\sqrt{\frac{8}{27}}(Z - X)^{\frac{3}{2}}\right] \dot{X}\dot{Z} dX dZ + \sum_{j=2}^{4} dx_j^2\right\}, \tag{2.16}$$

where one can check that whereas the singularity at $T = 0$ still remains, the metric is now regular at $T = 1$.

Since replacing W for Y in Eqs. (2.6) and (2.15) simultaneously leads to the condition $Y = -\frac{1}{2}\ln T + const.$, and hence, by the definition of Y, $Y = const$ and $W = const$ at $T = 1$, one can choose the singularities at $T = 1$ (i.e. at the initial time at which the brane system starts evolving along $T > 1$) to nest chronology horizons in the five-space. So, CTC's would only appear in the bulk.

B. Physical implications

If one considers a quantum field propagating in our spacetime, then the renormalized stress-energy tensor $\langle T_{\mu\nu}\rangle_{ren}$ would diverge at the chronology horizons [28]. The existence of this semiclassical instability would support a chronology protection also against the existence of our universe model. However chronology protection can be violated in situations which use an Euclidean continuation and lead to a vanishing renormalized stress-energy tensor everywhere, even on the chronology horizons. In order to convert metric (2.16) into a positive definite metric, it is covenient to use new coordinates p, q, defined by $X = p - q$, $Z = p + q$, or $T^2 = \exp\left[(4q/3)^{3/2}\right]$, $W^2 = 4p^2q/3$. A positive definite metric is then obtained by the continuation $p = i\xi$ which, in turn, implies $W = i\Omega$. Furthermore, using Eqs. (2.5) we can also see that this rotation converts the extra direction ω in pure imaginary and keeps t and T real, while making the first two of these three quantities periodic and leaving T unchanged. Two *ansätze* can then be used to fix the value of P_Ω, the period of Ω in the Euclidean sector. On the one hand, from the expression $\exp(W) \to \exp(i\Omega)$ we obtain $P_\Omega = 2\pi$, a result that allows us to introduce a self-consistent Li-Gott vacuum [29], and hence obtain $\langle T_{\mu\nu}\rangle_{ren} = 0$ everywhere. On the other hand, if we take $\exp(p) \to \exp(i\xi)$, then we get $P_\Omega = 2\pi \ln^{1/3} T^2$. In this case, for an automorphic scalar field $\phi(\gamma X, \alpha)$, where γ represents symmetry (2.3), α is the automorphic parameter [30], $0 < \alpha < 1/2$, and $X = t, \omega, x^2, x^3, x^4$, following the analysis carried out in [31,32], one can derive solutions of the field equation $\Box\phi = \Box\bar{\phi} = 0$ by demanding t-independence for the mode-frequency. This amounts [32] to a quantum condition on time T which, in this case, reads $\ln T^2 = (n+\alpha)^3 \ln T_0^2$, where T_0 is a small constant time. The use of this condition in the Hadamard function leads to a value for $\langle T_{\mu\nu}\rangle_{ren}$ which is again vanishing everywhere [32]. This not only solves the problem of the semiclassical instability, but can also regularize expression (2.14) at $T = 0$ and $T = 1$:

$$\rho_b + 3p_b = \frac{2T_0^{-2(n+\alpha)^3}\left(1 + \frac{1}{3(n+\alpha)^3 \ln T_0}\right)}{\kappa_{(4)}^2 \cosh^2(W)(n+\alpha)^3 \ln T_0}, \tag{2.17}$$

which can never diverge if we choose the constant T_0 such that $\ln T_0 \neq 0$.

In order to extract some physical consequences from the Misner-symmetry based brane model discussed so far, one would compare our five-dimensional metric (2.6) with the solution obtained by Randall and Sundrum [4,5] which reads:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad \mu,\nu = 0,1,2,3, \tag{2.18}$$

where $\eta_{\mu\nu}$ is Minkowski metric and $0 \leq y \leq \pi r_c$ is the fifth coordinate. Actually, metric (2.6) can be written in the same form as for metric (2.18) if we re-define coordinates T and W such that

$$\tau = \int \frac{\sqrt{\dot{T}^2 - T^2\dot{W}^2}}{T\ln^{2/3} T^2}dT, \quad r_c y = \int \frac{\sqrt{\dot{T}^2 - T^2\dot{W}^2}}{\ln^{2/3} T^2}dW, \tag{2.19}$$

where $\tau = \int d\eta/a(\eta)$ is the conformal time relative to four-space. On each brane defined at $W = W_0$ we can readily integrate the first of Eqs. (2.19) to yield $\tau = 3\ln^{1/3} T^2/(2\cosh W_0)$, that is we re-obtain for the scale factor the consistent value $a(\tau) = \frac{2}{3}\cosh W_0 \tau$ that describes a radiation dominated universe in terms of conformal time. We also notice that on surfaces of constant $T = T_c$, the second of Eqs. (2.19) can also be integrated to give

$$r_c y = \frac{i}{2\ln^{1/3} T_c^2} \ln\left(\frac{\sqrt{T_c^2 + \omega^2} - T_c}{\sqrt{T_c^2 + \omega^2} + T_c}\right),$$

which in turn implies $\sinh W \sinh\left(r_c y \ln^{1/3} T_c\right) = \pm 1$, and hence we have that on such constant surfaces

$$|y| = \frac{1}{r_c \ln^{1/3} T_c^2} \ln\left[\coth\left(\frac{W}{2}\right)\right], \tag{2.20}$$

meaning that W tends to infinity (zero) as y goes to zero (infinity). Actually, using the coordinate re-definitions (2.19), metric (2.6) can be written as

$$ds^2 = \ln^{2/3} T^2 \eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2, \tag{2.21}$$

where $0 \leq y \leq \pi$. This shows invariance under brane permutation, $\omega = 0 \leftrightarrow \omega = \omega_c$, as the two branes are defined for the same value of T, $T \geq 1$. One can then introduce the gravitational action for our Misner-brane model to be:

$$S_{\rm grav} = 2M^3 \int d^4x \int dy \sqrt{-g^{(5)}} R^{(5)}.$$

The four-dimensional graviton zero mode follows from metric (2.21) by replacing the Minkowski metric $\eta_{\mu\nu}$ for a four-dimensional metric $\bar{g}_{\mu\nu}(x)$. It is described [5] by an effective action following from taking $y = r_c y$ and substitution in the action

$$S_{\rm grav} = 2M^3 r_c \int d^4x \int dy \ln^{2/3} T^2 \bar{R}. \tag{2.22}$$

From this action we can derive the Planck mass [5]

$$M_p^2 = 2M^3 \int_0^{\pi r_c} dy \ln^{2/3} T^2, \quad (2.23)$$

or using $\exp(-2ky) = \ln^{2/3} T^2$ and replacing πr_c for y in the upper integration limit,

$$M_p^2 = \frac{M^3}{k}\left(1 - \ln^{2/3} T^2\right), \quad (2.24)$$

which reduces to $M_p^2 = M^3/k$ at $T = 1$, the initial moment (big bang) of the evolution of the Misner-brane universe. Thus, the initial time at $T = 1$ corresponds to taking the brane at $\omega = \omega_c$ away to infinite in the second Randall-Sundrum model [5]. As the Misner-brane universe then slowly evolves along $T > 1$ values, M_p^2 will decrease according to Eq. (2.24) while the distance between the two branes along ω decreases, as dictated by Misner symmetry. It follows that the velocity of expansion of the observable universe must be proportional to the speed at which the two branes approach to one another when Misner symmetry is satisfied.

On the other hand, as expressed as a non-relativistic problem, the Klein-Gordon equation for small gravitation fluctuations h in the Misner-brane case can be written as [5]

$$\left[-\frac{1}{2}\partial_z^2 + V(z)\right]\Phi(z) = m^2\Phi(z), \quad (2.25)$$

where

$$V(z) = \frac{15z^2}{8(k|z|+1)^2} - \frac{3k}{2}\delta(z), \quad (2.26)$$

with the new coordinate z defined by

$$z = \frac{1}{k}\left(\left|\ln^{-1/6} T^2\right| - 1\right)$$

$$\Phi(z) = \ln^{-1/6} T^2 \Phi(y) \quad (2.27)$$

$$h(x,z) = \ln^{-1/6} T^2 h(x,y)$$

In Misner-brane cosmology the use of Eqs. (2.27) allows us to obtain for the wave function of the bound-state graviton zero mode ($m = 0$):

$$\Phi_0(T) = \frac{1}{k}\sqrt{|\ln T^2|}. \quad (2.28)$$

We note that $\Phi_0(1) = 0$, $\Phi(T > 1) > 0$ and $\Phi(0) = \infty$. This means that the graviton is here strongly localized at the farthest possible distance from the branes along the fifth coordinate, and that it can only physically influence the four-dimensional universe once this has started to evolve, but not at the moment of its creation at $T = 1$. This result might be interpreted by considering that the structure of spacetime at just the moment of big bang is not influenced by quantum gravity.

An additional tower of continuous $m \neq 0$ KK (Kaluza-Klein) modes [5] also exist whose wave functions are given in terms of Bessel functions J and Y [33]. For the Misner-brane scenario these wave functions are

$$\Phi_C(T) = \left(\frac{1}{k}\left|\ln^{-1/3} T^2\right|\right)^{1/2} C_2\left(\frac{m}{k}\sqrt{\left|\ln^{-1/3} T^2\right|}\right), \quad (2.29)$$

with $C = J, Y$. Far from the branes, as T approaches 0, $\Phi_Y \to \infty$ and $\Phi_J \to 0$, while near the branes, as T approaches 1, the wave functions reach an oscillatory regime:

$$\Phi_C(T \to 1) \simeq \sqrt{\frac{2}{\pi m}} S\left(\frac{m}{k}\sqrt{\left|\ln^{-1/3} T^2\right|} - \frac{5}{4}\pi\right), \quad (2.30)$$

with $S = \sin$ if $C = Y$ and $S = \cos$ if $C = J$. It is interesting to notice that all the above wave functions satisfy the boundary condition

$$\partial_z \Phi_\jmath(z)|_{T=1} = 0, \quad (2.31)$$

where $\jmath = 0, C$. One would regard Eq. (2.31) as expressing the initial condition for the gravitational quantum state of the Misner-brane universe, and note that it is equivalent to the boundary condition on the regulator brane at $y_c = \pi r_c$ when $r_c \to \infty$, in the second Randall-Sundrum model [5].

We note finally that if the T-quantization $\ln T^2 = (n+\alpha)\ln T_0^2$ is adopted, then the argument of the Bessel function in wave function (2.29) becomes

$$\sqrt{\left|\frac{m^2}{k^2(n+\alpha)\ln^{1/3} T_0^2}\right|}.$$

It would then follow that the graviton mass is quantized in units of $\ln^{1/3} T_0^2$, so that

$$m^2 = k^2(n+\alpha)\ln^{1/3} T_0^2. \quad (2.32)$$

In this case, one would replace the gravitonless singularity at the big bang for an initial quantum state describing a graviton characterized by a minimal nonzero mass $m_0^2 = k^2\alpha \ln^{1/3} T_0^2$.

Before closing this section, it is convenient to make clear some points of the Miner-brane model where an incomplete interpretation could lead to misundertanding. Thus, at first sight, it could seem that Misner symmetry describes simple and familiar spacetimes. Specifically one would believe this by showing that Misner symmetry converts the five-dimensional metric (2.1) into merely a reparametrization of the Kasner-type solution [34]. However, the simple transformation

$$Q = \frac{1}{2}(f+g) = \ln T, \quad X = \frac{1}{2}(g-f) = W$$

converts metric (2.1) into

$$ds^2 = (2Q)^{-2/3} \dot{u}\dot{v} \left(-dQ^2 + dX^2\right) + (2Q)^{2/3} \sum_{j=2}^{4} dx_j^2,$$

which differs from a Kasner-type metric by the factor $\dot{u}\dot{v} = 1 - |\dot{\omega}|^2$ in the first term of the right-and-side. This factor cannot generally be unity in the five-dimensional manifold. On surfaces of constant $W = W_0$, according to Eqs. (2.5), we have $\dot{\omega} = \tanh W_0$, so $\dot{u}\dot{v} = \cosh^{-2} W_0$ which can only be unity for $W_0 = 0$ that is on the brane at $\omega = 0$. However, as it was pointed out in the introductory paragraph of this section, besides identifying the two branes according to Eq. (2.7), the Misner approach also requires that the closed up direction ω contracts at a given nonzero rate $d\omega_c/d\eta = -v_0$ [23]. This in turn means that once the branes are set in motion toward one another at the rate v_0, symmetry (2.7) should imply that for constant W_0,

$$\frac{dW_0(0)}{d\eta} = 0 \leftrightarrow \frac{dW_0(\eta)}{d\eta} - nv_0 = 0,$$

so that $dW_0(\eta)/d\eta \neq 0$ if $n \neq 0$. In this case, we have $\triangle W_0(\eta) = n \int_0^\eta d\omega_c = n \triangle_\eta \omega_c$, and hence $W_0(\eta) = W_0(0) + n \triangle_\eta \omega_c = n \triangle_\eta \omega_c > 0$, provided that we initially set $W_0 \equiv W_0(0) = 0$. It follows that $\dot{u}\dot{v}$ can only be unity on the brane at $\omega = 0$ when $n = 0$ (i.e. at the very moment when the brane universe was created and started to evolve. We note that if we substract the zero-point contribution $\alpha \ln^{1/3} T_0^2$, the quantization of T discussed above amounts to the relation $\eta \propto n^2$ and, therefore, initial moment at $\eta = 0$ means $n = 0$), taking on smaller-than-unity values thereafter, to finally vanish as $\eta, n \to \infty$. Thus, one cannot generally consider metric (2.1) or metrics (2.6) and (2.8) to be reparametrizations of the Kasner solution neither in five nor in four dimensions, except at the very moment when brane at $\omega = 0$ starts being filled with radiation, but not later even on this brane.

We note that in the case that Kasner metric would exactly describe our spacetime (as it actually happens at the classical time origin, $T = 1$, $n = 0$), Misner identification reduces to simply identifying the plane $W = X = 0$ with $W = X = n\omega_c$, that is identifying W on a constant circle, which does not include CTC's. This picture dramatically changes nevertheless once n and η become no longer zero, so that $\dot{u}\dot{v} = \cosh^{-2} W_0 < 1$ and the metric cannot be expressed as a reparametrization of the Kasner metric. In that case, there would appear a past apparent singularity [actually, a past event (chronology) horizon] at $T = 1$ for observers at later times $\eta, n \neq 0$, which is extendible to encompass nonchronal regions containing CTC's, as showed before by using the extended metric (2.16). Indeed, the particular value of T-coordinate $T = 1$ measures a quantum transition at which physical domain walls (three-branes) with energy density ρ_b created themselves, through a process which can be simply represented by the conversion of the inextendible physical singularity of Kasner metric [34] at $T = 1$, $n = 0$ into the coordinate singularity of the Misner-brane metric at $T = 1$, relative to observers placed at later times $\eta, n \neq 0$, which is continuable into a nonchronal region on the bulk space.

On the other hand, since the energy density ρ and pressure p on any of the two candidate branes vanish, one might also think that, related to the previous point, we are actually dealing with a world with no branes, but made up enterely of empty space. The conversion of the field-equation term (2.13) in a stress-energy tensor would then simply imply violation of momentum-energy conservation. However, the existence of an event (chronology) horizon which is classically placed at $T = 1$ for the five-dimensional spacetime amounts to a process of quantum thermal radiation from vacuum, similar to those happening in black holes or de Sitter space [35,36], which observers at later times $\eta, n > 0$ on the branes would detect to occur at a temperature $\beta \propto \ln^{-1/3} T^2$, when we choose for the period of Ω (which corresponds to the Euclidean continuation of the *timelike* coordinate W on hypersurfaces of constant T) $P_\Omega = 2\pi \ln^{1/3} T^2 \propto a$. Thus, for such observers, the branes would be filled with radiation having an energy density proportional to $\ln^{-4/3} T^2 \propto \eta^{-2} = \rho_b$ and temperature $\propto \eta^{-1/2}$, i.e. just what one should expect for a radiation dominated universe and we have in fact obtained from Eq. (2.14). Observers on the branes at times corresponding to $T > 1$, $n \neq 0$ would thus interpret all the radiating energy in the four-dimensional Misner-brane universe to come from quantum-mechanical particle creation near an event horizon at $T = 1$.

Moreover, in order to keep the whole two-brane system tensionless relative to a *hypothetical* observer who is able to pass through it by tunneling along the fifth dimension (so that when the observer enters the brane at $\omega = 0$ she finds herself emerging from the brane at $\omega = \omega_c$, without having experienced any tension), one *must* take the tension $V_{\omega=0} = \rho_b > 0$ and the tension $V_{\omega=\omega_c} = -\rho_b$, and therefore the total tension experienced by the hypothetical observer, $V = V_{\omega=0} + V_{\omega=\omega_c}$ will vanish. Given the form of the energy density ρ_b, this necessarily implies that current observers should live on just one of the branes (e.g. at $\omega = 0$) and cannot travel through the fifth direction to get in the other brane (so current observers are subjected to chronology protection [37]), and that, relative to the hypothetical observer who is able to make that traveling, the brane which she emerges from (e.g. at $\omega = \omega_c$) must then be endowed with an antigravity regime with $G_N < 0$ [6,10], provided she first entered the brane with $G_N > 0$ (e.g. at $\omega = 0$).

III. BLACK HOLES ON THE BRANES

The branes can be defined as the hypersurfaces at ω=const., with the fifth direction given as $d\omega = n_\mu dx^\mu$, where n_μ is the vector unit normal to the four-manifold $\mathbf{B}$ (the brane), with x^μ the coordinates [6]. This definition implies [34]

$$a^\mu = n^\nu \nabla_\nu n^\mu, \tag{3.1}$$

so that the five-dimensional metric (2.6) can be assumed to be

$$ds^2 = q_{\mu\nu} dx^\mu dx^\nu + d\omega^2, \tag{3.2}$$

where $q_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ is the induced metric on the four-manifold $\mathbf{B}$. With this notation the gravitational field equations for the identified two Misner branes can be written as [6]:

$${}^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + 8\pi G_N \tau_{\mu\nu} + \kappa_{(5)}^4 \pi_{\mu\nu} - E_{\mu\nu}, \tag{3.3}$$

in which $\tau_{\mu\nu}$ is the energy-momentum tensor in the branes, and

$$\Lambda_4 = \frac{1}{2}\kappa_{(5)}^2 \left(\Lambda_5 + \frac{1}{6}\kappa_{(5)}^2 \lambda^2\right) \tag{3.4}$$

$$\pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\alpha}\tau_\nu^\alpha + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2 \tag{3.5}$$

$$G_N = \frac{\kappa_{(5)}^4 \lambda}{48\pi}, \tag{3.6}$$

with λ the brane tension.

In vacuum $\tau_{\mu\nu} = 0$, and hence $\pi_{\mu\nu} = 0$, so that from Eq. (3.3)

$${}^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} - E_{\mu\nu}. \tag{3.7}$$

If, following Dadhich, Maartens, Papadopoulos and Zezania [9], we choose the bulk cosmological constant such that it satisfies $\Lambda_{(5)} = -\kappa_{(5)}^2\lambda/6$ (or rather for Misner-brane cosmology, setting $\Lambda_{(5)} = 0$ and taking into account that the total brane tension is zero), then we have $\Lambda_{(4)} = 0$, so that

$${}^{(4)}G_{\mu\nu} = -E_{\mu\nu}, \tag{3.8}$$

where in general we have for a static vacuum

$$E_{\mu\nu} \propto U\left(u_\mu u_\nu + \frac{1}{3}h_{\mu\nu}\right) + P_{\mu\nu}, \tag{3.9}$$

with U an effective energy density on the brane which arises from the free gravitational field in the bulk, $P_{\mu\nu}$ the effective anisotropic stress coming also from the free gravitational field in the bulk, and $h_{\mu\nu} = q_{\mu\nu} + u_\mu u_\nu$, u_μ being the chosen four-velocity field.

Since if no further symmetry other than Z_2-symmetry is included U and P are generally nonzero, it was obtained in [9] that black holes in the brane at ω=0 has a Reissner-Nordstrom like metric, with the "tidal charge" arising from the gravitational effects of the fifth dimension playing the role of an "electric charge" which does not exist physically. This rather nonconventional behaviour of black holes may be regarded as the counterpart in static spherically symmetric vacuum of the nonlinear right-hand-side terms that appear in the field equations that correspond to the homogeneous and isotropic flat brane cosmology [7]. If we impose nevertheless Misner symmetry $\hat{M}$ to the induced four-metric $q_{\mu\nu}$, then we have

$$\hat{M} q_{\mu\nu} = q_{\mu\nu}, \tag{3.10}$$

and in this case $q_{\mu\nu}$ would no longer be arbitrary. This amounts to vanishing values for the quantities U and P. Thus, if $q_{\mu\nu}$ is made to satisfy Misner symmetry, $E_{\mu\nu} = 0$ and the field equations (3.7) strictly correspond to the vacuum case:

$${}^{(4)}G_{\mu\nu} = 0. \tag{3.11}$$

If, in addition to Misner symmetry, we further impose spheric symmetry and staticness to the induced metric $q_{\mu\nu}$, we then obtain either the usual Schwarzschild solution

$$-q_{tt} = \frac{1}{q_{RR}} = 1 - \left(\frac{2M}{M_p^2}\right)\frac{1}{R}, \tag{3.12}$$

in the case that we choose $\Lambda_{(5)} = 0$, or the Schwarzschild-anti de Sitter (or -de Sitter) solution otherwise.

In order to re-express the four-dimensional Misner-brane metric (2.8) as a Schwarzschild line element, one will introduce first spheric symmetry in coordinates x_j's (i.e. $x_j \to r, \phi, \theta$), and then the change of variables

$$R = r\ln^{1/3} T^2 = \frac{r}{2\gamma_R(1+\gamma_R)} \tag{3.13}$$

$$t = \frac{\ln^{1/3} T^2}{2\cosh W_0}\left[\ln^{1/3} T^2 + \frac{3}{4}\gamma_R\left(2\ln^{1/3} T^2 - 1\right)\right]$$

$$= \frac{1 + \frac{3}{2}\gamma_R\left[1 - \gamma_R(1+\gamma_R)\right]}{8\cosh W_0 \gamma_R^2 (1+\gamma_R)^2}, \tag{3.14}$$

where

$$\gamma_R = \sqrt{1 - \frac{2M}{R}}, \tag{3.15}$$

with M the black hole mass. We finally note that the new coordinates R and t depend on T and r only. The

four-dimensional Misner-brane metric for vanishing cosmological constant in the bulk becomes

$$ds^2 = -\gamma_R^2 dt^2 + \frac{dR^2}{\gamma_R^2} + R^2 d\Omega_2^2, \tag{3.16}$$

in which $d\Omega_2^2$ is the metric on the unit two-sphere. Thus, the past and future singularities correspond to $T=1$, the event horizon to $T=\infty$, and the spatial infinity to $T=\exp(1/96)$. In this way, we have succeeded in obtaining conventional black holes in the Misner-brane framework. One should interpret this as a proof of the consistency of that framework.

IV. THE QUANTUM STATE OF THE MISNER-BRANE UNIVERSE

In this section we will calculate the wave function of the Misner-brane universe by using the semiclassical approach to the Euclidean path-integral formalism [38]. The boundary initial condition for this universe should be that it was not created from nothing like in the no boundary or tunneling conditions [39], but created itself [40] as corresponds to the spacetime slicing at sections with $W=W_0$=const. of a five-dimensional manifold whose fifth direction is filled with CTC's. Such an initial condition is already incorporated in the four-dimensional metric given by Eq. (2.8). This can be written

$$ds^2 = -\frac{dT^2}{T^2\cosh^2 W_0 \ln^{2/3} T^2} + \ln^{2/3} T^2 ds_3^2, \tag{4.1}$$

where ds_3^2 is the three-dimensional flat metric.

The path integral would be given as:

$$\psi\left[h_{ij}\right] = \int_C d\mu\left[g_{ab}\right]\exp\left\{iS\left[g_{ab}\right]\right\},$$

where $S\left[g_{ab}\right]$ is the Hilbert-Einstein Lorentzian action for the Misner-brane universe with four-dimensional metric g_{ab}, $a,b=0,1,2,3$ and C is the class of four-geometries which are bounded by a given hypersurface on which they induce the set of data h_{ij}, $i,j=1,2,3$, predicted by Misner-like symmetry (2.3). To obtain a manageable expression for the wave function one should make the rotation $W=i\Omega$, keeping T real, and then introduce the semicalssical approximation [39,41] to yield:

$$\psi \sim \exp\left[-S_E(A)\right],$$

with a the scale factor. The Hilbert-Einstein action containing the necessary boundary term [38] is

$$S(T) = -\frac{1}{16\pi G_N}\int d^3x \int_1^T dT'\sqrt{-g(T')}R(T')$$

$$+\frac{1}{8\pi G_N}\int d^3x TrK(T)\sqrt{h(T)}, \tag{4.2}$$

in which TrK is the trace of the second fundamental form, and g and h are the determinants of the four-metric and the induced metric on the boundary, respectively. From metric (4.1) we have for the scalar curvature, R, and the extrinsic curvature, K,

$$R = -\frac{8}{3}\frac{\cosh^2 W_0}{\ln^{4/3} T^2} \tag{4.3}$$

$$TrK = 2\frac{\cosh W_0}{\ln^{2/3} T^2}. \tag{4.4}$$

Inserting Eqs. (4.3) and (4.4) into the action (4.2) and integrating, we obtain

$$S(T) = \frac{M_p^2 V_3}{2\pi}\cosh W_0 \ln^{1/3} T^2, \tag{4.5}$$

where $M_p = (8\pi G_N)^{-1/2}$ is the four-dimensional Planck mass and $V_3 = \int d^3x$ is the three-volume. In terms of the cosmological time $\eta = 3\ln^{2/3} T^2/(4\cosh W_0)$, action (4.5) becomes:

$$S(\eta) = \frac{M_p^2 V_3}{\sqrt{3}\pi}\cosh^{3/2} W_0 \eta^{1/2}. \tag{4.6}$$

The total action will be the sum of action (4.5) [or (4.6)] at $\omega=0$ and at $\omega=\omega_c$. In the Euclidean formalism where T is kept real and $W=i\Omega$, if we set $W_0=0$ and take the Li-Gott type period $P_\Omega = 2\pi$, and hence a unique Ω-separation between the branes of 2π, the action on the two branes turns out to be

$$S_E(T)_{\omega=0} = -S_E(T)_{\omega=\omega_c} = \frac{M_p^2 V_3}{2\pi}\ln^{1/3} T^2, \tag{4.7}$$

because the Newton constant changes sign on the brane at $\omega=\omega_c$ [6,10] (see also Sec. II), and we have taken into account the Misner identification of the two branes and the orbifold symmetry. Hence, if we consider a Li-Gott type self-consistent vacuum, the total Euclidean action $S_E^{\rm total} = S_E(T)_{\omega=0} + S_E(T)_{\omega=\omega_c} = 0$ and therefore the semiclassical probability for the Misner-brane universe will be

$$P = \psi^2 \sim e^{-2S_E^{\rm total}} = 1. \tag{4.8}$$

According to the discussion in Sec. II, however, in order to avoid the singular character of both the renormalized stress-energy tensor and the initial moment of the universe, instead of a Li-Gott type Euclidean period $P_\Omega = 2\pi$ [29], an Euclidean period $P_\Omega = 2\pi\ln^{1/3} T^2 = 2\pi(n+\alpha)\ln^{1/3} T_0^2$ should be used. In this case, however, there is an ambiguity in the choice of Ω-separation between branes. Taking into account once again Misner and orbifold symmetries, so as the feature that cosmological time $\eta \propto (n+\alpha)^2$ implies an upper bound $\sqrt{\eta}$ for the possible values which the integer n may take on, in

this case, one should in fact place the two branes at any Ω-separation given by $\triangle\Omega = 2\pi(n'+\alpha)\ln^{1/3}T_0^2$, where $n' = 0,1,2,...,n$. Assuming a normalization of the minimum time such that $T_0^2 = e$, we would have then

$$\triangle\,\Omega = 2\pi(n'+\alpha) \leq P_\Omega, \quad n' = 0,1,2,...,n, \qquad (4.9)$$

and hence we get for the total Euclidean action

$$S_E^{\rm total}(n)$$

$$= \frac{M_p^2 V_3}{\pi}(n+\alpha)\sin^2(\pi\alpha) \equiv \frac{M_p^2 V_3}{\pi}a(\eta)\sin^2(\pi\alpha), \quad (4.10)$$

which is defined in terms of n through the scale factor $a(\eta)$, but does not depend on n', and hence on brane separation. Thus, the ambiguity in the choice of brane separation does not manifest in the final expression for the wave function representing the quantum state of the Misner-brane universe, a fact that physically suffice to ensure the necessary invariance of the quantum state.

The above normalization of the minimum time T_0 is rather conventional. It has been introduced merely for the sake of simplicity in the equations and will be kept in what follows unless otherwise stated. We note that this normalization implies that if we assume any particular value or constraint on the parameter α, we are actually assuming that particular value or constraint on the quantity $\alpha\ln^{1/3}T_0^2$. On the other hand, the total action would only become zero whenever the automorphic parameter α [30] is zero. However, α is defined so that it can only take on nonzero values which are smaller than 1/2. Moreover, if this parameter vanished, then the stress-energy tensor would diverge on the η-time ground state $n = 0$ (see Sec. II). Moreover, if as required by Misner symmetry we set the branes into a mutually approaching motion at a Lorentzian speed β [23] whose absolute value is directly related to the velocity with which galaxies are receding from one another (see Sec. II), then there must exist a linear relation between the three-volume V_3 and the Lorentzian volume on the extra dimension $V_1 = \pi\ell_c$. Taking this relation to be $V_1 M_p^{-2} = \pi V_3$, from the known expression $M_p^2 = M_{(5)}V_1$ [4,5] we finally achieve the condition

$$\pi V_3 M_{(5)}^3 = 1. \qquad (4.11)$$

Inserting condition (4.11) into the Euclidean action (4.10) we obtain

$$S_E^{\rm total} = \frac{M_p V_1}{\pi^2}(n+\alpha)\sin^2(\pi\alpha). \qquad (4.12)$$

Thus, for $\alpha << 1$, we can approximate $S_E^{\rm total} \simeq M_p V_1 \alpha^3$ on the ground state $n = 0$. In this case, the probability for the universe would be very large not just in the ground state, but also on the low-lying excited states. The ansatz $V_1 M_p^{-2} = \pi V_3$ can in fact be related with the Hubble law as follows. Since action (4.12) does not depend on spacelike directions, the continuity equation for the Misner-brane universe should read for $\alpha << 1$

$$\frac{dP}{d\eta} = 0, \quad P = \exp\left(-2M_p V_1 a(\eta)\alpha^2\right), \qquad (4.13)$$

in which $a(\eta)$ is the scale factor. From expressions (4.13) it inmediately follows

$$\beta \equiv v_0 = -\frac{dV_1}{d\eta} = V_1 H, \qquad (4.14)$$

where v_0 is the speed at which the Misner-like circumference $2\pi r_c$ (with r_c being the radion [42]) is contracting (see Sec. II), and $H = \dot{a}/a$ is the Hubble constant. Eq. (4.14) is in fact the Hubble law for the extra direction.

V. THE COSMOLOGICAL QUANTUM METRIC

We turn now to study the possible effects that the extra dimension may cause on the metric of the four-dimensional Misner-brane universe. In principle, these effects can be of two different types. On the one hand, propagation along the fifth direction will causally modify the spacetime on the branes and, on the other hand, communication between the two branes or different regions on each of these branes can be carried out through processes involving CTC's that shortcut spacetime along the fifth direction. All of these effects can be quantum-mechanically described by using the Euclidean path-integral formalism [38]. Actually, it would be natural to calculate the quantum-mechanically disturbed four-dimensional metric as that effective metric which results from including all the above-mentioned effects from the fifth dimension in the four-dimensional Misner-brane metric (2.8) by propagating the quantum state of the undisturbed metric through the fifth direction from the position of one brane to the position of the other. This propagator can typically be given as a path integral of the form:

$$\langle g_{ab}, \omega = 0 | g_{ab}, \omega = \omega_c \rangle = \int d\mu[g_{\mu\nu}] \exp\left\{iS[g_{\mu\nu}]\right\}, \qquad (5.1)$$

where a,b=0,1,...,3, μ,ν=0,1,...,4, $d\mu$ is the integration measure, and $S[g_{\mu\nu}]$ is the five-dimensional Lorentzian Hilbert-Einstein action which is given by

$$S[g_{\mu\nu}] = -\frac{1}{16\pi G_{(5)}}\int d^3x \int_0^{W_c} dW \int dT \sqrt{-g^{(5)}}R^{(5)}$$

$$+ \frac{1}{8\pi G_{(5)}}\int d^3x \int_0^{W_c} dW Tr K_{(4)}\sqrt{h^{(4)}}, \qquad (5.2)$$

with $W_c = \sinh^{-1}(\omega_c/T)$ the value of the extra W-coordinate at ω_c, $G_{(5)}$ the five-dimensional gravitational

constant, $g^{(5)}$ and $h^{(4)}$ the determinants of the five-dimensional metric and the spacelike metric induced on the boundary whose second fundamental form is $K_{(4)}$, respectively, and $R^{(5)}$ the five-dimensional scalar curvature.

Such as it is expressed by Eqs. (5.1) and (5.2) this path integral does not have any clear physical significance because the values $\omega = 0$ and $\omega = \omega_c$ (and hence $W = 0$ and $W = W_c$) merely express coordinate labels. This should be related to the well-known fact that classical general relativity does not allow the existence of well-defined degrees of freedom [34]. However, the kind of "quantization" of the fifth dimension [induced by Misner identification of branes at these values of ω together with its "quasiharmonic" discretization (see Sec. II)] which comes in addition to the quantization implied by the usual notion of path integral, allows us to take such ω values as the true physical quantities that determine the dynamics and actually the very existence of the universe itself. Taking into account this additional quantization, so as Misner identification of the two branes and orbifold symmetry, the physically meaningless path integral (5.1) can be in fact converted into the physical quantity:

$$\langle g_{ab}, W=0 | g_{ab}, \triangle W\rangle =$$

$$\int d\mu[g_{\mu\nu}]\Big|_{W=W_0} \exp\left\{ iS[g_{\mu\nu}]|_{W=W_0}\right\}, \qquad (5.3)$$

with $\triangle W = i\triangle\Omega$ the brane separation in the Lorentzian sector, and

$$S[g_{\mu\nu}]|_{W=W_0} =$$

$$-\frac{\triangle W}{16\pi G_{(5)}}\int d^3x \int dT \sqrt{-g^{(5)}|_{W=W_0}}\left(R^{(5)}\Big|_{W=W_0}\right)$$

$$+\frac{\triangle W}{8\pi G_{(5)}}\int d^3x \left(TrK_{(4)}|_{W=W_0}\right)\sqrt{h^{(4)}|_{W=W_0}}, \qquad (5.4)$$

where the quantities $X|_{W=W_0}$ are the same as X but evaluated at $W = W_0$ =const. once they have been computed using W as a variable.

To obtain a manageable, physical expression of this propagator we now make [38] an Euclidean rotation with $W = i\Omega$ and the physically most interesting brane separation $\triangle W = i\triangle\Omega = 2\pi i(n'+\alpha)$, so that the integral (5.3) becomes an integral over $\exp\left\{ -S_E[g_{\mu\nu}]|_{\Omega=\Omega_0}\right\}$, with Ω_0 =const. In the semiclassical approximation where

$$|\psi_{ab},\Omega\rangle \to \psi\left(g_{ab},\Omega\right) \qquad (5.5)$$

$$\tilde{g}_{ab} \equiv g_{ab}|_{\Omega=\Omega_0} \to \frac{\delta}{\delta\pi^{ab}}\left\{\ln\left[\psi\left(a_{ab},\Omega\right)\right]\right\}, \qquad (5.6)$$

with $\psi\left(g_{ab},\Omega\right)$ a quasiclassical wave function and $\pi^{ab} = \ln^{-4/3} T^2$ the momentum conjugate to g_{ab}, we obtain for the wave function

$$\psi\left[g_{ab}, 2\pi(n'+\alpha)\right] \sim \psi\left[g_{ab},0\right]\exp\left\{ -S_E\left[g_{\mu\nu}\right]|_{\Omega=\Omega_0}\right\}, \qquad (5.7)$$

from which one can obtain an approximate expression for the four-dimensional effective metric in terms of the cosmological time containing the effects from the fifth dimension by using the operation (5.6). It is worth noticing that the metric eigenstate (5.7) does depend on brane separation, a fact which one should expect as it expresses the already mentioned relation between universal expansion and variation of brane separation.

The Euclidean action can now be evaluated from the five-dimensional metric components. These first produce

$$R^{(5)}\Big|_{W=W_0} = \frac{8}{3}\frac{\cosh^2 W_0}{\ln^{\frac{4}{3}} T^2} \qquad (5.8)$$

$$TrK^{(4)}\Big|_{W=W_0} = -\frac{4}{3}\frac{\cosh^2 W_0}{\ln^{\frac{2}{3}} T^2}, \qquad (5.9)$$

and then inserting expressions (5.8) and (5.9) into the action (5.4), performing the integration over T, rotating $W = i\Omega$, and finally setting $W_0 = i\Omega_0 = 0$ (without loss of generality) and $\triangle\Omega = 2\pi(n'+\alpha)$ as in Sec. IV, it is obtained for the Euclidean action:

$$S_E = -\frac{V_3(n'+\alpha)}{8G_{(5)}}\ln\left(\ln T^2\right), \quad n' = 0,1,2,...,n. \qquad (5.10)$$

The disturbed quasiclassical wave function becomes then

$$\psi\left[g_{ab}, 2\pi(n'+\alpha)\right] \sim \psi\left[g_{ab},0\right]\exp\left\{\frac{V_3(n'+\alpha)}{8G_{(5)}}\ln\left(\ln T^2\right)\right\}, \qquad (5.11)$$

which depends on both n' and n, the latter dependence taking place through $\ln T$. Using the definition $8\pi G_{(5)} = M_{(5)}$ and the condition (4.10), we can obtain the eigenvalues of the disturbed effective metric $\tilde{g}_{ab}$ by applying operation (5.6) to the wave function (5.11). We get

$$\tilde{g}_{ab} = g_{ab} + (n'+\alpha)\ln^{\frac{4}{3}} T^2, \qquad (5.12)$$

where $g_{ab} = \ln^{2/3} T^2$ is the classical metric. Taking into account that $\sqrt{\eta} = (n+\alpha)$, one can finally express Eq. (5.12) in terms of the cosmological time η as:

$$\tilde{g}_{ab} = g_{ab} + (n'+\alpha)\eta^2 = g_{ab} + (n'+\alpha)(n+\alpha)^4, \qquad (5.13)$$

with the integer number $n' = 0,1,2,..,n$ the same as that was introduced in Eq. (4.9), and we have disregarded here and hereafter dimensionless factors of order unity in

the expressions for a, g_{ab} and h_{ab}. Thus, depending on the values of $\eta(n)$, α and n', two regimes can be distinguished for expression (5.13): (i) $\eta >> 1$, $n' > 0$ where $\tilde{g}_{ab} \simeq (n'+\alpha)\eta^2$ which reaches a maximum value $\eta^{5/2}$ at $n' = n$, and (ii) $\eta << 1$ (i.e. n=0) where the second term in the right-hand-side of Eq. ((5.13) can be regarded as a perturbation, h_{ab}, to metric g_{ab}; in the extreme case that $n' = n = 0$, we get a minimum value for this perturbation $h_{ab} = \eta^{5/2}$. In the following subsections we shall discuss some cosmological consequences from the Misner-brane model in these two extreme cases.

A. Solution to the horizon and flatness puzzles

Misner-brane cosmology cannot accommodate any realistic model of inflation and therefore contends with it as an explanation for the physics in the early universe. You can readily convince yourself of this incompatibility if you consider that for inflation to take place in a brane world it is necessary that the absolute value of the tension on the two involved branes be different [15], a requirement which is both conceptually and operationally incompatible with Misner symmetry, as is the necessity for brane inflationary models [15] that functions $f(u)$ and $g(v)$ be both exponential.

Nevertheless, the existence of one extra direction makes inflation unnecessary to solve the horizon and flatness puzzles. Even for a fully causal behaviour along the extra dimension, Chung and Frees have shown [17] that signals traveling along null geodesics on extra space may connect distant points which otherwise are outside the four-dimensional horizon. If spacetime tunneling is moreover allowed to occur, then additional two-way transmissions of signals between spacelike separated regions would occur that permitted such regions to come into thermal contact [43]. Misner-brane cosmology combines these two mechanisms and, therefore, by itself provides a solution to the standard cosmological puzzles.

A more technical argument comes from the form of metric (5.13) in the extreme case of regime (i) where $n' = n >> 1$, i.e. $\eta >> 1$. Here, the scale factor for the radiation-dominated universe is given by $\tilde{a}(\eta) = \eta^{5/4}$. Thus, after the Planck era, we have $\dot{\tilde{a}}(\eta) \equiv d\tilde{a}(\eta)/d\eta = \eta^{1/4}$, and hence

$$\dot{\tilde{a}}(\eta_{\rm exit}) > \dot{\tilde{a}}(\eta_1), \quad \eta_{\rm exit} > \eta_1, \tag{5.14}$$

where $\eta_{\rm exit}$ is the time at the radiation-matter transition, and η_1 is an early time which still occurs much later than Planck era. The inequality (5.14) clearly solves the cosmological horizon puzzle [44]. Since one can always write this inequality as the equality

$$\dot{\tilde{a}}(\eta_{\rm exit}) = \beta\dot{\tilde{a}}(\eta_1), \quad \beta > 1,$$

the ratios of the energy density to the critical densities $\Omega_{\rm exit}$ and Ω_1 can be related by

$$\beta^2 \left|\Omega_{\rm exit} - 1\right| = \left|\Omega_1 - 1\right|, \tag{5.15}$$

so implying that $|\Omega - 1|$ need not be initially set to a very small value in order to insure small later values. This also provides with a solution to the flatness puzzle [44] because β can take on very large values even during the radiation-dominated era, so that $|\Omega_{\rm exit} - 1|$, and hence the current value $|\Omega_0 - 1|$, get on very small values without resorting to any fine tuning.

B. Origin of density perturbations

Perhaps the greatest success achieved by inflationary models so far be their prediction of a scale-invariant spectrum for density perturbations [45]. Such a success cannot however hide the rather dramatic feature that all realistic models of inflation considered so far fail to predict a reasonably small amplitude for the perturbation spectrum [46]. In the absence of any inflationary mechanism, Misner-brane cosmology should by itself produce primordial density perturbations which, in this case, must be generated by quantum fluctuations of the spacetime metric itself, rather than any matter field. This inmediately connects with the extreme regime (ii) with $\eta << 1$, $n' = n = 0$ for the disturbed metric (5.13). In this regime, $g_{ab} = \eta$ and $h_{ab} = \eta^{5/2}$. The fluctuation spectrum will be given as a Fourier transform of h_{ab} given by

$$h_k = 3\sqrt{\frac{2}{\pi}}\int_0^{\pi/k_*^{ab}} h^{ab}\exp\left(ig_{ab}k^{ab}\right)dg_{ab}, \tag{5.16}$$

where the momenta k_*^{ab} and k^{ab} are mutually related by $k^{ab} = a(\eta)k_*^{ab}$ for k-modes that enter the Hubble radius, i.e. $k^{ab} = 2\pi Ha(\eta)$ at $\eta = \eta_{\rm exit}$ [46]. Inserting $h^{ab} = \sqrt{\eta}$ and $g_{ab} = \eta$ in Eq. (5.16) and performing the integral after noticing that the horizon size is $H^{-1} = 2\eta$, we obtain for $n' = n = 0$ and $\alpha << 1$

$$k^{3/2}h_k \simeq 3\sqrt{\frac{2\alpha}{\pi}}. \tag{5.17}$$

For this spectrum, the contribution of each interval from k to γk ($\gamma > 1$) to the total dispersion,

$$D = 4\pi\int_k^{\gamma k} dk k^2\left|h_k^2\right| \simeq 72\alpha\ln\gamma, \tag{5.18}$$

is scale-independent, as required by the Zeldovich-Harrison theory [47] and is also predicted by realistic inflationary models [45].

We turn now to consider density perturbations and their spectrum. For metric perturbations of the type given by Eq. (5.13), the density contrast of the perturbations of matter density (or temperature) is given by [48]

$$\left(\frac{\delta\rho}{\rho}\right)^{ab}=\frac{a\dot{a}\dot{h}^{ab}-h^{ab}}{3\left(1+\dot{a}^2\right)}\simeq\frac{1}{3}\sqrt{\eta}, \tag{5.19}$$

where $a=\sqrt{\eta}$. The amplitude of the spectrum of density fluctuations will be then

$$\delta_k=3\sqrt{\frac{2}{\pi}}\int_0^{\pi/k_*^{ab}}\left(\frac{\delta\rho}{\rho}\right)^{ab}\exp\left(ig_{ab}k^{ab}\right)dg_{ab}. \tag{5.20}$$

Inserting expression (5.19) in Eq. (5.20) and performing the integral transform we finally obtain for $n'=n=0$

$$k^{3/2}\left|\delta_k\right|\sim\sqrt{\frac{2\alpha}{\pi}}\ln^{1/6}T_0^2, \tag{5.21}$$

where we have restored, for the moment, a generic value for T_0. This is one of the main results of this work. It states that in Misner-brane cosmology the density of perturbations has the scale-invariant spectrum with enough a small amplitude. From the bounds on the anisotropy of CMB we know [49] that $\sqrt{\alpha}\ln^{1/6}T_0^2$ should then be smaller than 10^{-4}, with possibly a value between 10^{-5} and 10^{-6}. This is quite a reasonable bound on $\alpha\ln^{1/3}T_0^2$ that, in the present model, is also compatible with the perturbative character of h_{ab} (needing $\ln^{5/3}T_0^2\alpha^5 << g_{ab} << 1$ for $n'=n=0$), and corresponds to a high probability for the universe (see Sec. IV). On the other hand, it was seen in previous sections that the lowest possible bound for both α and $\ln T_0^2$ can never be zero. We note furthermore that perhaps the presence of baryons in the perturbations would place the minimum size in such a way that the minimum and therefore most probable value of the quantity $\alpha^{1/2}\ln^{1/6}T_0^2$ be close to the required value between 10^{-5} and 10^{-6}. It is in this sense that the results obtained so far for Misner-brane cosmology conform better to experiment than those obtained from realistic models of inflation which all imply too large an amplitude for density fluctuations [45,46]. In the next subsection, it will be seen that the higher-dimensional cosmological approach considered in this paper seems to produce better predictions than inflation also for the spectrum of CMB anisotropies.

C. CMB anisotropies

The fluid-dynamical theory underlying primordial fluctuations of background temperature [50] predicts the existence of small-angle CMB anisotropies coming from the primeval ripples, first observed by COBE [19], at the recombination time. These anisotropies correspond to sound waves and show a power spectrum that carries precise fundamental information about the origin of fluctuations and the fate of the universe. There are two current paradigms able to predict the power spectrum of CMB anisotropies, inflation [51] and topological defects, particularly cosmic strings and textures [52]. Whereas realistic models of inflationary cosmology predict a power spectrum which is "superluminarly" generated by perturbations that exceeded the horizon size after crossing it at a common time, and contains a fundamental mode at sky angle $\theta\simeq 1^\circ$ followed by a succession of more or less harmonic secondary coherent peaks at lower angles (which imply a nearly topologically flat universe and a time-coherent creation of the sound waves), the so-called causally generated oscillations, e.g. by cosmic string network, were the result of fluctuations with sizes well inside the horizon which never crossed it, and lead to a power spectrum whose fundamental mode is shifted to angles smaller than $\theta\simeq 1^\circ$ and whose secondary peak structure is destroyed by temporal incoherence induced by causal random forcing of the oscillators and ausence of common horizon crossing.

Reliable observations of the power spectrum of CMB anisotropies have only been made quite recently. Boomerang [20] and Maxima [21] results seem now able to allow some discrimination between the above contending models. Thus, the coincidence of a first peak at $\theta\simeq 1^\circ$ in both experiments point clearly in favour of inflation (see however Refs. [53,54]). However, although the results seem clearly in favour of whichever models where the perturbations crossed the horizon to exceed it thereafter until last scattering, it is by no means so clear that they may declare inflation as the only particular mechanism which is responsible for the observed power spectrum. In fact, the two experiments also agree in that the amplitude of at least the second peak at $\theta\simeq 0.35^\circ$ is dramatically smaller than that is predicted by inflation [55].

Inflation appears then to be in trouble when trying to explain not only the amplitude of density fluctuations estimated by COBE [19], as pointed out in Subsec. V.B, but also the spectrum of CMB anisotropies measured by Boomerang [20] and Maxima [21]. Thus, the time could be up for the emergence of new alternatives where perturbations were also forced to cross the horizon. In what follows we show that Misner-brane cosmology might be one of such alternatives. While the perturbations in our approach keep the "good" property of horizon crossing, they possess two novel properties which are worth remarking and might be of much interest to justify experiment.

Firstly, since metric perturbations in this case can generally be written as

$$h_{ab}=(n'+\alpha)\eta^2=(n'+\alpha)(n+\alpha)^4, \tag{5.22}$$

with $n'\leq n$, fluctuations will cross the horizon not just at a common time $\eta_c\simeq 1$, for $n'=n=1$, but also during a following time interval $\triangle\eta$ which is characterized by integral numbers such that $n'<n$. On the brane at $\omega=0$, which has positive Newton constant $G_N>0$, the perturbations smaller than the horizon size will then be causally washed out eventually, so that one should expect the interval for the time-incoherent horizon crossing $\triangle\eta$ to be small, though nonzero. The resulting situation

would match what is predicted by inflation, except for the involved time-incohence induced by a nonzero horizon-crossing time interval which would partially destroy the coherent structure of the secondary peaks. Secondly, on the brane with $G_N < 0$ at $\omega = \omega_c$, only those fluctuations which are inside the horizon ($n' < n$) are able to keep the oscillations stable. This is caused by inversion of the gravitational forces and hence radiation pressure in the wells [56]. Fluctuations exceeding the horizon size will then be inexorably washed out. However, even sub-horizon fluctuations on the brane at $\omega = \omega_c$ will eventually cross the horizon and then inexorably dissipate out, except possibly for the residual fraction with the smallest values of n', whenever the automorphic parameter α and the minimum time T_0 take on values that satisfy $\alpha \ln^{1/3} T_0^2 \leq \eta_*^{-1}$, with η_* being the recombination time. This constraint on $\alpha \ln^{1/3} T_0^2$ is compatible with the amplitude of the density contrast considered in Subsec V.B. Fluctuations surviving inside the horizon at recombination on the brane with negative G_N for small n' will also produce oscillations with fundamental mode at $\theta \simeq 1^\circ$, and secondary peaks whose structure is destroyed by temporal incoherence induced by causal random forcing of the oscillators.

In summary, Misner-brane cosmology predicts quantum fluctuations satisfying evolution equations in the radiation-dominated regime of the form [50,57]:

$$[\Theta + \Psi](t_*) = [\Theta + \Psi](0)\cos(ks) \tag{5.23}$$

$$v_\gamma = \sqrt{3}[\Theta + \Psi](0)\sin(ks), \tag{5.24}$$

where $\Theta = \delta T/T$, Ψ is the Newtonian potential, $t_* = \int_0^{\eta_*} d\eta/a(\eta)$ is the conformal time at last scattering, and $s = \int_0^{t_*} c_s dt$ is the sound horizon at last scattering, with c_s the sound speed and $\Theta = \mp\Psi/2$, with the choice of sign depending on whether perturbations on the brane at $\omega = 0$ (upper sign) or on the brane at $\omega = \omega_c$ (lower sign) are considered. Since Misner symmetry identifies the two branes, relative to an hypothetical observer who is able to travel through the fifth direction, the perturbations on both branes should simultaneously contribute the power spectrum for CMB anisotropies [50,57]. Thus, relative to that observer and $\omega = 0$, this spectrum becomes:

$$C_\ell \simeq \frac{2}{\pi}\sum_{\omega=0}^{\omega_c}\int \frac{dk}{k}k^3\left[(\Theta_\omega + \Psi_\omega)\,\jmath_\ell(kd) + v_{\gamma\omega}\jmath'_\ell(kd)\right]^2, \tag{5.25}$$

in which the $\jmath_\ell$'s are spherical Bessel functions of the first kind [33], and $d = t_0 - t_*$, with t_0 the current conformal time.

For current observers who only are able to observe what is happening on just the brane at $\omega = 0$, the power spectrum will reduce to

$$C_\ell \simeq \frac{2}{\pi}\int \frac{dk}{k}k^3\left[(\Theta_{\omega=0} + \Psi_{\omega=0})\,\jmath_\ell(kd) + v_{\gamma\omega=0}\jmath'_\ell(kd)\right]^2. \tag{5.26}$$

Obviously, the spectrum given by Eq. (5.26) produces the same fundamental ($\ell \simeq 180/\theta \simeq 200$) and overtone ($\ell \simeq 500$, etc) acoustic modes as in the spectrum predicted by simplest inflationary models [], but with the rather remarkable difference of having quite smaller heights for the secondary peaks. Although this prediction seems to be quite compatible with Boomerang and Maxima results, it appears that any conclusion will only be attainable from the future results provided by MAP [58] and Planck surveyor [59] at lower angles.

VI. CONCLUSIONS

Within the spirit of the Randall-Sundrum approach, we have considered a five-dimensional apacetime whose metric satisfies Misner symmetry, discussing the cosmological and gravitational implications arising from the resulting brane-world model. After reviewing the Misner-brane universe and analysing some of its physical characteristics, a Misner-brane black hole scenario has been constructed in which the conventional metric of a Schwarschild or anti-de Sitter black hole is obtained when neutral matter in the branes is allowed to collapse without rotating. By using then a semiclassical approximation to the Euclidean path-integral approach, we have calculated the quantum state of the Misner-brane universe and the quantum effects induced on its metric by brane propagation along the extra coordinate. It has been seen that Misner symmetry requires that the absolute value of the tension in the two branes be always the same, and hence it follows that our brane scenario is fully incompatible with the existence of any inflationary period in the radiation dominated era. However, since communications between distant regions which are outside the horizon can still be done both causally and by means of CTC's through the fifth dimension, the horizon and flatness problems can be solved in our model.

Perhaps the most remarkable results in this paper be the predictions of a scale-independent spectrum of density fluctuations whose amplitude can, contrary to inflation, be easily accommodated to the existing observational bounds. Density fluctuations come here about as a result from the existence of metric perturbations on the branes induced by the above-alluded brane propagation along the extra direction, at the earliest cosmological times. At later times, these perturbations grow beyond the horizon during a nonzero time interval on the two branes and give rise therefore to a power spectrum of CMB anisotropies whose acoustic peaks are at exactly the same small sky angles as in the spectrum corresponding to the simplest inflationary models, but with secondary peaks whose intensity is expected to be greatly

diminished with respect to the inflation-generated spectrum. This prediction seems to conform to recent measurements by Boomerang and Maxima better than those from inflationary models do.

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